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BOOK REVIEW

Renormalization of Quantum Field Theory
[Review of the book; "QED : A Proof of Renormalizability"]
(Lecture Notes in Physics, Vol. 312, 1st edn)
edited by J S Feldman, T R Hurd, L Rosen and J D Wright

Springer-Verlag · Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong, 1988
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The ultraviolet divergence problem is the most basic problem in local quantum field theory and this has not yet been solved. String theory which may eventually replace local quantum field theory as the fundamental theory of the physical universe, offers the hope of a finite theory in which the divergences are eliminated altogether. String theory is basically a nonlocal field theory which necessarily includes gravitation and in this theory Planck mass plays the role of the regulator for the divergences. However, string theory is still far from a complete theory and so, for the present we have to live with local quantum field theory with all its divergences.

The ultraviolet divergences of local quantum field theory actually arise from summing over virtual processes occurring at infinite energies. Thus, in principle, a knowledge of the processes at infinite energies enters into the calculation of any physical quantity relevant even at low energies. This coupling between low and infinite energies, which is a basic characteristic of any local relativistic quantum field theory would seem to make it a theory of doubtful value. For, how can one supply the requisite information on processes occurring at infinite energies? To add to the troubles, the contribution from the infinite energies actually turns out to be infinite!

Fortunately, there exists a class of local quantum field theories called renormalizable theories which yield finite answers to physical quantities inspite of the presence of divergences in the intermediate steps of the calculation. Further, this finite answer is also independent of information on the processes and interactions occurring at arbitrarily high energies of which we are ignorant.

Quantum electrodynamics (QED) was the first successful renormalizable quantum field theory. But, it took many years and the heroic efforts of many people to chart a safe course through the many technical problems of renormalization theory (severe combinatorial and graphical complexities as well as the notorious problem of "overlapping divergences"). To quote the authors, "Some of the milestones in this journey were the original work on QED by Feynman, Schwinger and Tomonoga, the refinements by Dyson, Mathews and Salam,.. the Dyson-Weinberg Power Counting Theorem, the renormalization prescription of Bogoliubov and Parasiuk and the subsequent improvements of Hepp and Zimmerman, culminating in the 1960's in what is now known as BPHZ renormalization".

With the advent of the renormalizable electroweak theory and the renormalizable quantum chromodynamics (QCD), the importance of renormalization theory has increased considerably. Now, it is not only electromagnetism but the whole of high energy physics that is described by renormalizable quantum field theory. This is the standard model comprising electroweak theory and QCD. Computation of higher-order effects in the renormalizable standard model has been pursued with increasing vigour in the recent years and the results are being used to confront the high-precision experimental tests of the standard model.

Without detracting from the importance of these calculations one must admit that these higher-order calculations are infact a mess. In the context of QED in 1950's Feynman had remarked that the problem with the higher-order calculations is that, having calculated it to some order α, you do not learn anything about the result in next order; you cannot even guess the sign correctly. Electroweak theory and QCD being more complex (nonabelian gauge fields, spontaneous breakdown of symmetry, unknown Higgs sector, too many parameters...), the situation is much worse. It is therefore not surprising that more than 100 authors have contributed to the subject so far and more than 1500 papers have been written, but there is still no intuitive understanding.

It is in this context that the book under review assumes importance. A beginning has been made in the application of Wilson's renormalization group ideas to renormalization theory. Renormalization group may help in the charting of a path in the forest of perturbative renormalization calculations. This approach pioneered by Gallavotti and coworkers, is based on making scale decompositions of fields : $\Phi = \Sigma_{h=-\infty}^{\infty} \Phi^{(h)}$, where $\Phi^{(h)}$ has length scale M^{-h}, $M > 1$ being a fixed scale parameter. By successively integrating out the fields $\Phi^{(h)}$ (from high to low h), Gallavotti and Nicolo obtained a natural and beautiful tree expansion. According to the authors of the book, "The GN tree expansion dramatically simplifies the problem of perturbative renormalization, enabling one to make a choice of counter terms and to renormalize scale by scale without ever seeing overlapping divergences or the usual combinatorial complexities".

In this monograph, Feldman *et al* provide a complete exposition of this route to renormalization theory and apply the method to the proof of renormalizability of QED We now give a detailed section-wise summary.

In Sec.1, the scale decomposition of the fields and of the photon and electron propagators D and S is introduced. Also the counter terms, Ward identities and the auxiliary regularization for S are discussed.

Sec. 2 which contains the GN tree expansion and ultraviolet renormalization is the heart of the whole programme. In fact, if we ignore gauge invariance and infrared divergence which are the two special problems of QED, the proof of ultraviolet renormalizability of QED is already given in this section. The subsequent sections complete the proof by taking into account of gauge invariance and infrared divergence.

There exists a basic conflict between the GN procedure and gauge invariance. The electron-propagator with cut-off defined through scale decomposition does not preserve gauge invariance and the resulting Ward identity is violated. This problem is solved by the authors by invoking the well-known Pauli-Villars regularization and Sec. 3 is devoted to this.

In Sec. 4, it is proved that for renormalized QED, with photon cut offs U and I (ultraviolet and infrared) and loop regularizati(. (Pauli-Villars) Λ, Ward identities hold and the counter terms are gauge invariant.

In Sec. 5, the existence of the limits $\Lambda \to \infty$ and $U \to \infty$ are proved. Combining this with the results of Sec. 2, the existence of the double limit is guaranteed.

In Sec. 6, which deals with the infrared divergence problem, a tree expansion for a general field theory involving massless fields is developed.

Finally in Sec. 7, the proof of renormalizability of QED is completed. This is done by using the results of Sec. 6 to show the existence of QED in perturbation theory in the infrared limit $I \to -\infty$, and to extend the results of Sec. 3–5 (which applied only when $I = 0$), thus proving the gauge invariance of the renormalization.

Sec. 8 devoted to local Borel summability, offers a bouns. A bound on the large-order behaviour of perturbation theory for a broad class of models is proved. In particular, this bound implies that QED is locally Borel summable.

Appendix A provides a list of symbols and terminology which enhances the usefulness of the book.

Since the treatment in the text exclusively deals with Euclidean spacetime (*i.e.* imaginary time), appendix B explains how renormalization is carried out in real time.

The authors have done a remarkable job in presenting the reader with a coherent view of this new approach to renormalization, but the book is certainly not a "pleasure cruise". The authors refer to the famous "Salam criterion" for renormalization : "The difficulty, as in all this work, is to find a notation which is both concise and intelligible to at least two persons, of whom one may be the author". Possibly the proof of renormalizability of QED given by Feldman *et al* satisfies the Salam criterion. But the present reviewer must confess that he has not yet qualified as that other person who is the guarantor of the criterion.

As examples of the nontrivial notational complication, consider the notations in the book :

$$V^{U}_{U,un}\left(\Phi^{\le U}\right) \quad \text{in eq. (2.1)}$$

$$V^{U,\Lambda,N}_{k,un}\left(\Phi^{\le k}\right) \quad \text{in eq. (3.14).}$$

Perhaps such a complication is an intrinsic characteristic of renormalization theory.

A more basic criticism is that unfortunately the regularization by scale decomposition turns out to be not sufficient for the gauge-invariant renormalization of QED. The authors are forced to introduce the Pauli-Villars regularization also as an auxilliary regularization.

Gauge invariance may prove a even bigger obstacle for the renormalization of nonabelian gauge theories *via* the scale-decomposition method. No mention has been made by the authors, as to whether and how this approach could be generalized to these cases. In view of the fact that the present-day Standard Model of High Energy Physics is based on nonabelian gauge fields, this could be considered a serious omission.

In spite of the above critical remarks, the authors deserve praise for having undertaken the task of exposing this new approach to renormalization theory to a wider audience. This will serve as a significant step in the evolution of renormalization theory. The book is recommended for all serious students of quantum field theory.

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